NUMERICAL STUDY OF COUPLED HEAT TRANSFERS THROUGH A VERTICAL CAVITY WITH ALVEOLAR WALLS

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In this work, we study numerically the two dimensional coupled heat transfers through a honeycomb structure formed by a vertical cavity separating two alveolar walls. Heat transfers are assumed to be two-dimensional and the air motion in all cavities of the system is laminar. The left and right vertical sides of the hollow structure are considered isothermal. The top and bottom horizontal sides are adiabatic. Equations governing natural convection in the cavities, heat exchange by radiation between the surfaces of the different cavities and heat conduction in the solid partitions are solved by the SIMPLE algorithm. Effects of convection and radiation on the linearity of the global heat transfer through the system are studied. Overall heat exchange coefficients for the hollow structure are derived based on the simulation results.

Keywords: honeycomb structure, vertical cavity, coupled heat transfers, conduction, convection, radiation, numerical simulation

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Introduction

The honeycomb structures intervene in several thermal systems. In particular, they are used very currently in the construction of building walls because of the advantages that they present on the material and energetic plans. The prediction of the heat flow through such building components using the analytical transfer functions methods is not possible because of the non-linearity of the heat transfers by convection and radiation in the alveolar of hollow blocks.

The heat transfer within such structures is done simultaneously by conduction in the different solid partitions, by natural convection inside the cavities and by radiation between the internal faces of the last ones. These three heat transfer processes are intimately bound. Therefore, a fine study of the thermal behavior of the hollow blocks needs a simultaneous resolution of the complex and non linear equations modeling the different mechanisms of heat transfer.

However, the available studies in the literature are generally limited to simple configurations consisting in rectangular cavities with one or several conducting walls. Earlier investigations were conducted by Balvanz and Kuehn [1] and Kim and Viskanta [2] on the interaction between the natural convection in a square cavity and the heat conduction in the adjacent walls. Effects of surface radiation on natural convection in a square enclosure filled with air were studied by Balaji and Venkateshan [3, 4], Akiyama and Chong [5], Ramesh and Venkateshan [6] and Ramesh et al. [7]. In these studies, it has been shown that natural convection heat transfer is significantly reduced by conduction in the walls and/or radiation exchange between the cavity surfaces. Coupled heat transfers by conduction, natural convection and radiation in cellular structures with two vertical series of square cavities has been studied numerically by Abdelbaki and Zrikem [8]. Application was presented for building walls made of hollow clay tiles. Later, numerical solution of combined heat transfers in hollow clay tiles, with two air cells deep, submitted to transient thermal excitations was performed by Abdelbaki et al. [9]. Based on the simulation results the authors derived empirical transfer function coefficients (TFC) for the hollow clay tiles by applying an identification technique. It should be noted that such TFC cannot be derived using analytical or semi-analytical algorithms available in the literature [10, 11].

In the present work, we study numerically the two dimensional coupled heat transfers through a honeycomb structure formed by a vertical cavity separating two alveolar walls. Analysis of the flow structures and the temperature fields in the different alveolar is presented. The influence of the non linearity of convection and radiation heat transfer on the global heat transfer through the honeycomb structure is studied. Finally, appropriate overall heat exchange coefficients are determined.

Mathematical formulation

The geometry of the two dimensional configuration to be studied is presented in Fig. 1. It represents a honeycomb structure of width L and height H formed by a vertical cavity confined with air and separating two cellular walls. The width and height of the vertical cavity are respectively l and h. Each cellular wall is formed by a vertical range of Ny rectangular alveolar of width l' and height h'. The total numbers of cavities of he studied honeycomb structure in x and y directions are respectively Nx and Ny. The different cavities are surrounded by vertical solid partitions of thickness ex_i ($1 \le i \le 4$) and horizontal ones of thickness ey_j ($1 \le j \le Ny + 1$).

For the thermal boundary conditions of the problem, the left and right vertical sides of the honeycomb structure are considered isothermal and are maintained at constant temperatures T_o and T_i respectively. The top and bottom horizontal sides are assumed to be adiabatic.

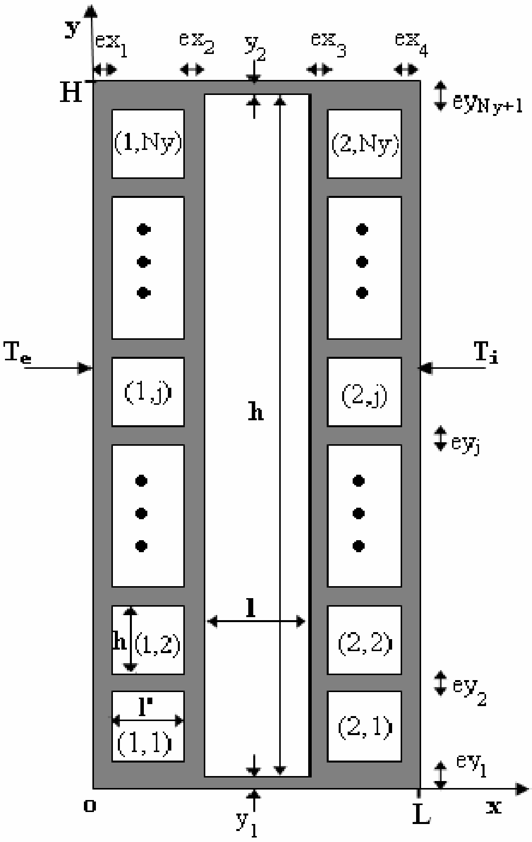


Fig. 1. *Schematic diagram of the studied honeycomb structure*

In formulating governing equations, the fluid motion and the heat transfer are considered to be two-dimensional and laminar. The solid and fluid properties are assumed to be constant except for the density in the buoyancy term where the Boussinesq approximation is utilized. Viscous heat dissipation in the fluid is neglected. The fluid is assumed to be non-participating to radiation and the cavities inside surfaces are considered diffuse-grey. Dimensionless equations governing the conservation of mass, momentum and energy for the air in the internal cavities are given by:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0\,, \tag{1}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right), \tag{2}$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}= =-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\cdot Pr\cdot\theta_f\,, \tag{3}$$

$$\frac{\partial \theta_f}{\partial \tau}+U\frac{\partial \theta_f}{\partial X}+V\frac{\partial \theta_f}{\partial Y}=\frac{\partial^2 \theta_f}{\partial X^2}+\frac{\partial^2 \theta_f}{\partial Y^2}\,, \tag{4}$$

where U, V, P and θ_f are the dimensionless variables associated, respectively, with the velocity components in X and Y directions respectively, the pressure, and the fluid temperature, Pr is the Prandtl number and Ra is the Rayleigh number given by: $Ra=\frac{g\beta L^3(Te-Ti)}{\nu^2}Pr$, $Pr=\frac{\nu}{\alpha_f}$, where ν and α_f are respectively the fluid kinematic viscosity and the thermal diffusivity.

The dimensionless equation of heat conduction in the solid walls is:

$$\frac{\alpha_f}{\alpha_s}\frac{\partial \theta_s}{\partial \tau}=\frac{\partial^2 \theta_s}{\partial X^2}+\frac{\partial^2 \theta_s}{\partial Y^2}\,, \tag{5}$$

where α_s is the solid thermal diffusivity and θ_s is the dimensionless solid temperature. The boundary conditions of the problem are:

* $U = V = 0$ on the inner sides of each cavity.
* $\theta_s(0,Y)=1$ *and* $\theta_s(1,Y)=0$ $(0\le Y\le A=H/L)$

$$\left.\frac{\partial \theta_s}{\partial Y}\right|_{Y=0}=\left.\frac{\partial \theta_s}{\partial Y}\right|_{Y=A}=0 \quad * \; (0\le X\le 1).$$

The continuity of the temperature and the heat flux at the fluid-solid interfaces gives:

$$\theta_s(X,Y)=\theta_f(X,Y)\,, \tag{6}$$

$$-\frac{\partial \theta_s}{\partial \eta}=-N_k\frac{\partial \theta_f}{\partial \eta}+N_r Q_r\,, \tag{7}$$

where η represents the dimensionless coordinate normal to the wall, N_k is the thermal conductivity ratio K_f/K_s, Q_r is the dimensionless radiative heat flux and N_r is the dimensionless radiation to conduction parameter defined by: $N_r=\frac{\sigma T_e^4 L}{k_s(T_e-T_i)}$.

The dimensionless radiative heat flux Q_r is related to the radiative heat flux q_r by: $Q_r=\frac{q_r}{\sigma T_e^4}$.

The net radiative heat flux $q_{r,k}(r_k)$ exchanged by the finite area dS_k, located at a position r_k on the surface k, is given by:

$$q_{r,k}(r_k)=J_k(r_k)-E_k(r_k)\,, \tag{8}$$

where $J_k(r_k)$ is the radiosity and $E_k(r_k)$ is the incident radiative heat flux on the surface dS_k given respectively by:

$$J_k(r_k)=\varepsilon_k\sigma(T_k(r_k))^4+(1-\varepsilon_k)E_k(r_k)\,, \tag{9}$$

$$E_k(r_k)=\sum_{j=1}^{4}\int_{A_j}J_j(r_j)\,dF_{dS_k-dS_j(r_k,r_j)}\,, \tag{10}$$

where ε_k is the emissivity of the surface k and $dF_{dS_k-dS_j}$ is the view factor between the finite surfaces dS_k and dS_j located at r_k and r_j respectively. Taking into account equations (8) to (10), the dimensionless radiative heat flux can be expressed as:

$$Q_{r,k}(r'_k)=\varepsilon_k(G-1)^4\left(\theta_k(r'_k)+\frac{1}{G-1}\right)^4- -\varepsilon_k\sum_{j=1}^{4}\int_{S_j}J'_j(r'_j)\,dF_{dS_k-dS_j}\,, \tag{11}$$

where G is the temperature ratio T_e/T_i, $J'_j(r'_j)$ is the dimensionless radiosity at the position r_j on surface j. By dividing the walls into finite isothermal surfaces, equation (11) leads to a set of linear equation where the unknowns are the dimensionless radiosities $J'_j(r'_j)$.

The dimensionless average heat flux across the structure is given by:

$$Q_a=-\frac{1}{A}\int_0^A\left.\frac{\partial \theta_s}{\partial X}\right|_{X=0}dX=-\frac{1}{A}\int_0^A\left.\frac{\partial \theta_s}{\partial X}\right|_{X=1}dX \tag{12}$$

The previous equations are discretized using the finite differences method based on the control volumes approach with a power law scheme and are solved by the SIMPLE algorithm. The resulting system of algebraic equations is solved by the Tri-Diagonal-Matrix-Algorithm. To accelerate the convergence of solutions, the governing equations are solved in their instationary form. The numerical code had been tested in previous studies [8, 9, 12]. A study on the effects of both grid spacing and time step on the simulation results has been conducted. The compromise between accuracy and computation time is found for a 75×91 non-uniform grid with a 16×16 non-uniform grid in each small cavity and 29×85 in the big cavity. The dimensionless time used in the simulation is 10^{-4}. The convergence criterion is based on the relative changes in the variables U, V, P, θ and Q_r at the different nodes of the calculation domain:

$\left|\dfrac{f^{n+1}(i,j)-f^{n}(i,j)}{f^{n}(i,j)}\right| \le 10^{-5}$, where $f^{n}(i, j)$ is the variable f value at node (i, j) calculated in the iteration n.

Results and discussion

Results presented in this study are obtained for structures having the geometrical parameters given in Table 1.The values of H and h depend on the number of alveolar in the vertical direction *(Ny)* and are calculated from the values of ey_j, y_1, y_2 and h'. The solid partitions thermal conductivity and emissivity are respectively $K_s = 1$ W/mK and $\varepsilon = 0.8$. The dimensionless parameter Nr depends on the temperature difference $\Delta T = (T_e - T_i)$ that takes values between 5 °C and 40 °C in accordance with the practical conditions. The air thermal conductivity K_f is equal to 0.0262 W/mK and the Prandtl number is $Pr = 0.71$.

Table 1

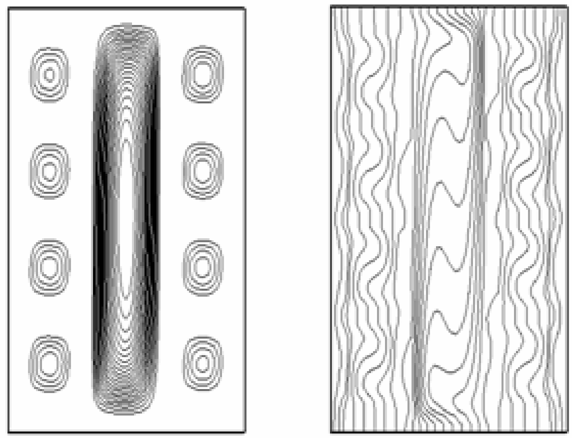
Geometrical dimensions of the different components of the honeycomb structure, cm

l	l'	h'	ex_i	ey_1	ey_i	ey_n	y_1	y_2
5	3,5	3,5	1	1,5	1	1,5	0,5	0,5

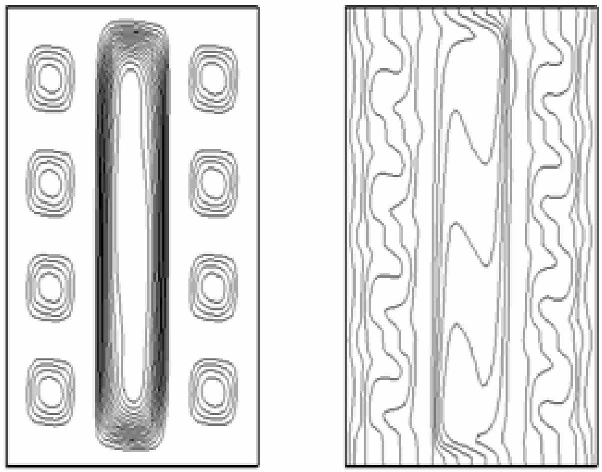
Streamlines and isotherms

Fig. 2 presents the streamlines contours (at the left) and the isotherms (at the right) obtained for a structure of $(Nx = 2)\times(Ny = 4)$ alveolar in addition to the big cavity and for the temperature differences $\Delta T = 5$ °C, $\Delta T = 20$ °C and $\Delta T = 40$ °C. The results of Fig. 2 show that the nature of the flow structures is characterized by a single cell turning clock-wise as well in the small alveolar that in the big cavity. As foreseen, the distortion of the streamlines in the big cavity becomes more pronounced when ΔT increases indicating an increase of the natural convection intensity. In fact, the values of the maximal stream function Ψ_{max} in the big cavity are 20.2, 25.3 and 28.8 for $\Delta T = 5$ °C, $\Delta T = 20$ °C and $\Delta T = 40$ °C respectively. The exam of the streamlines in the first vertical rows of alveolar shows that the size of the central cell deceases slightly when moving from the low cavity ($j = 1$) toward the one situated in top of the structure ($j = 4$) indicating a weak reduction of the intensity of the flow in this sense. This can be assigned to the interaction between the heat transfer by convection and radiation. This situation is reversed for the other vertical row of alveolar located at the right of the vertical cavity where the size of the flow intensity increases slightly from the cavity ($j = 1$) toward the cavity ($j = 4$).

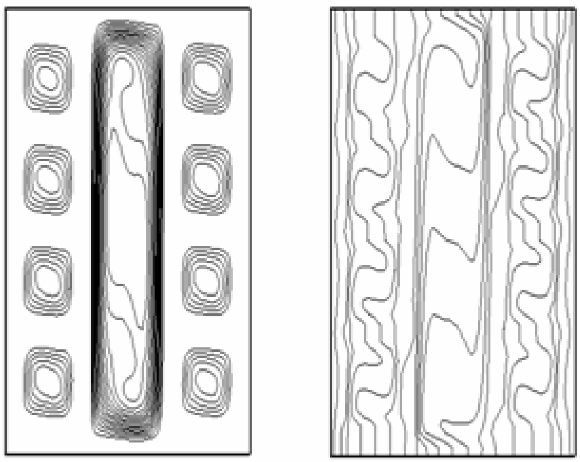
Concerning the temperature field, the distortion of the isotherms in the central regions of the different cavities reveals a very marked two dimensional heat transfer that becomes nearly unidirectional in the solid partitions separating the cavities where the isotherms are perpendicular to the main direction of heat transfer (direction ox). As expected, near the walls of the central cavity, the movement of air is faster and the gradients of temperature are more important. Then, the convective heat transfer is relatively important in these regions.



a: $\Delta T = 5$ °C



b: $\Delta T = 20$ °C



c: $\Delta T = 40$ °C

Fig. 2. *The streamlines contours (at the left) and the isotherms (at the right) obtained for a structure of $(Nx = 2)\times(Ny = 4)$ alveolar*

Heat transfer

In order to show the effect of the number of alveolar in the vertical direction (*Ny*) on the global heat transfer through the honeycomb structure, the Fig. 3 presents the variation of the dimensional heat flux crossing the structure Q (W/m^2) as a function of the temperature difference ΔT between the vertical sides of the latter. Fig. 3 gives the results obtained for different values of *Ny* using adiabatic boundary condition. As it can be seen, the differences between heat fluxes obtained for *Ny* = 4, 8 and 16 are negligible especially for temperature differences lower than 25 °C. Discrepancies that appear for ΔT superior than 25 °C are lower than 10 %. It should be noted that the global variation of Q as a function of ΔT is almost linear because of the predominance of the conduction heat transfer which represents more than 50 % of the overall heat transfer through the honeycomb structure.

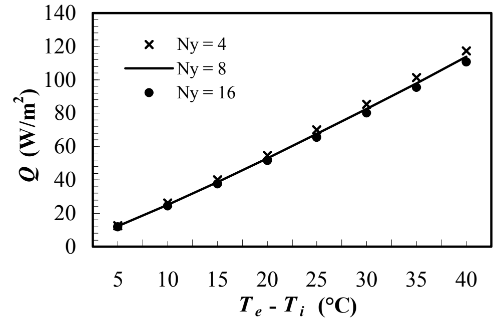


Fig. 3. *Effect of the alveolar number in the vertical direction Ny on the average heat transfer through the alveolar structures*

The linear behavior of Q with ΔT is very important because it permits to derive a overall heat exchange coefficient (overall conductance U) for the studied honeycomb structure. This overall conductance permits a fast and accurate prediction of the heat transfer through the system without solving the complex equations governing the heat transfer mechanisms which are coupled and locally non linear: $Q = (U{\cdot}\Delta T)$. For a honeycomb structure with *Ny* = 4, the overall conductance value obtained here is: $U_{present}$ = = 2.59 W/m^2. This value is markedly inferior to the overall conductance given in reference [12] ($U_{[12]}$ = 3.01 W/m^2) that corresponds to a hollow clay tile with three vertical ranges of alveolar constructed from the same material and having the same dimensions as the honeycomb structure treated here with *Ny* = 4. This result is expected because the central range of alveolar of the hollow block in reference [12] is replaced here by the vertical cavity. Then, the hollow clay tile studied in the present work permits a reduction of heat transfer about 15 % with respect to the hollow clay tile with three ranges of air cells deep mostly used in practice to construct building envelopes.

Conclusion

Coupled heat transfers by conduction, natural convection, and radiation in a vertical cavity with alveolar walls have been studied numerically. Analysis for the temperature differences that occur in practice shows that the flow structures in the different cavities are characterized by a single cell turning clock-wise. The variation of the number of alveolar of the vertical walls between *Ny* = 4 and *Ny* = 16 have not large effect on the global heat exchange through the honeycomb structure. The variation of the overall heat flux through the structure is found to be almost linear. Based on this result overall heat exchange coefficient had been derived for hollow clay tiles formed by a vertical cavity separating two alveolar walls. Also, it had been shown that the latter reduces considerably the heat transfer compared to the hollow clay tiles with three air cells in the horizontal direction which are mostly used in the construction of building envelopes.

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